# INTEGRATION
## BY PARTS
Facts to have on hand

| dv | v |
|---|---|
| $n$ | $nx$ |
| $x^n$ | $\frac{x^{n+1}}{n+1}$ |
| $\frac{1}{x}$ | $\ln\lvert x\rvert$ |
| $e^x$ | $e^x$ |
| $n^x$ | $\frac{n^x}{\ln(n)}$ |

| dv | v |
|---|---|
| $\sin x$ | $-\cos x$ |
| $\cos x$ | $\sin x$ |
| $-\cos x$ | $-\sin x$ |
| $-\sin x$ | $\cos x$ |
| $\cot x$ | $\ln\lvert\sin\rvert x$ |
| $\sec^2 x$ | $\tan x$ |
| $\csc^2 x$ | $-\cot x$ |

| | |
|---|---|
| $\ln x$ | $x \ln x - x$ |
| $\sec x$ | $\ln(\lvert\sec x + \tan x\rvert)$ |
| $\sec x \tan x$ | $\sec x$ |
| $\csc x$ | $\ln(\lvert\csc x + \cot x\rvert)$ |

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# Derivatives that show up regularly in Integration by Parts

| u | du |
|---|---|
| $x$ | $1$ |
| $nx$ | $n$ |
| $x^n$ | $nx^{n-1}$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $-\sin x$ | $-\cos x$ |
| $-\cos x$ | $\sin x$ |
| $\ln\lvert x\rvert$<br>$\ln x$<br>$\ln -x$ | $\frac{1}{x}$ |
| $\sec x$ | $\sec x \tan x$ |

| u | du |
|---|---|
| $n^x$ | $(\ln n)\, n^x$ |
| $\log_n x$ | $\frac{1}{x \ln n}$ |
| $e^x$ | $e^x$ |
| $\arcsin x$<br>aka $\sin^{-1} x$ | $\frac{1}{\sqrt{1-x^2}}$ |
| $\arccos x$<br>aka $\cos^{-1} x$ | $\frac{-1}{\sqrt{1-x^2}}$ |
| $\arctan x$<br>aka $\tan^{-1} x$ | $\frac{1}{x^2+1}$ |

aka means **Also Known As**

Note: this is *not* a complete list of derivatives & integrals, but it will get you started.

**Chain Rule:** $\frac{d}{dx} f(g(x)) = f'(g(x)) \bullet g'(x)$

**Product Rule:** $\frac{d}{dx}[f(x)\, g(x)] = f(x)g'(x) + g(x)f'(x)$

*Four Steps from Product Rule to IBP formula*

- Integrate all terms in the Product Rule:
  $f(x)g(x) = \int f(x)g'(x)dx + \int g(x)f'(x)$
- Rewrite in IBP friendly notation such that:
  $f(x) = u \qquad f'(x)\,dx = du$
  $g(x) = v \qquad g'(x)\,dx = dv$
- Rewritten: $uv = \int u\,dv + \int v\,du$
- Rearrange: $\int u\,dv = uv - \int v\,du$ Ta-da! IBP formula!
  (hey, remember to finish evaluations by attaching **+ C** to indefinite integrals)

| u | du |
|---|---|
| $2e^x$ | $2e^x$ |
| $e^{-x}$ | $-e^{-x}$ |
| $e^{3x}$ | $3e^{3x}$ |
| $e^{-1}x$ | $e^{-1}$ |
| $ex$ | $e$ |

| u | du |
|---|---|
| $x$ | $\frac{x^2}{2}$ |
| $x$ | $1$ |
| $x^2$ | $2x$ |
| $2x$ | $2$ |
| $n$ | $0$ |

Facts to have handy when evaluating $\int u\,dv = uv - \int v\,du$

| dv | v |
|---|---|
| $e^{2x}$ | $\frac{e^{2x}}{2}$ |
| $e^{-3x}$ | $-\frac{e^{-3x}}{3}$ |
| $\sin 4x$ | $-\frac{\cos 4x}{4}$ |
| $\cos 4x$ | $\frac{\sin 4x}{4}$ |
| $\sin 4x$ | $-\frac{\cos 4x}{16}$ |

| dv | v |
|---|---|
| $1$ | $x$ |
| $x$ | $\frac{x^2}{2}$ |
| $2x$ | $x^2$ |
| $x^2$ | $\frac{x^3}{3}$ |

Tabular Integration: $\int x^4 \sin x\,dx =$ ?

| *signs* | $u = x^4$ | $dv = \sin x\,dx$ |
|---|---|---|
| (+) | $x^4$ | $\sin x$ |
| (−) | $4x^3$ | $-\cos x$ |
| (+) | $12x^2$ | $-\sin x$ |
| (−) | $24x$ | $\cos x$ |
| (+) | $24$ | $\sin x$ |
| | $0$ | $-\cos x$ |

$= -x^4\cos x + 4x^3\sin x + 12x^2\cos x - 24x\sin x - 24\cos x + C$