

INTEGRATION

BY PARTS

Facts to have on hand

dv	v	dv	v
n	nx	sin x	-cos x
x ⁿ	$\frac{x^{n+1}}{n+1}$	cos x	sin x
$\frac{1}{x}$	ln x	-cos x	-sin x
e ^x	e ^x	-sin x	cos x
n ^x	$\frac{n^x}{\ln(n)}$	cot x	ln sin x
ln x	x ln x - x	sec ² x	tan x
sec x	ln(sec x + tan x)	csc ² x	-cot x
sec x tan x	sec x		
csc x	ln(csc x + cot x)		

Derivatives

that show up regularly in Integration by Parts

u	du	u	du
x	1	n ^x	(ln n) n ^x
nx	n	log _n x	$\frac{1}{x \ln n}$
x ⁿ	nx ⁿ⁻¹	e ^x	e ^x
sin x	cos x	arcsin x	$\frac{1}{\sqrt{1-x^2}}$
cos x	-sin x	aka sin ⁻¹ x	$\sqrt{1-x^2}$
-sin x	-cos x	arccos x	$\frac{-1}{\sqrt{1-x^2}}$
-cos x	sin x	aka cos ⁻¹ x	$\sqrt{1-x^2}$
ln x	$\frac{1}{x}$	arctan x	$\frac{1}{x^2+1}$
ln x	$\frac{1}{x}$	aka tan ⁻¹ x	$\frac{1}{x^2+1}$
ln -x	$\frac{1}{x}$		
sec x	sec x tan x		

aka means **Also Known As**

Note: this is *not* a complete list of derivatives & integrals, but it will get you started.

Facts to have handy when evaluating

$$\int u dv = uv - \int v du$$

dv	v	dv	v
e ^{2x}	$\frac{e^{2x}}{2}$	1	x
e ^{-3x}	$-\frac{e^{-3x}}{3}$	x	$\frac{x^2}{2}$
sin 4x	$-\frac{\cos 4x}{4}$	2x	x ²
cos 4x	$\frac{\sin 4x}{4}$	x ²	$\frac{x^3}{3}$
$\frac{\sin 4x}{4}$	$-\frac{\cos 4x}{16}$		

u	du
2e ^x	2e ^x
e ^{-x}	-e ^{-x}
e ^{3x}	3e ^{3x}
e ⁻¹ x	e ⁻¹
ex	e

u	du
$\frac{x^2}{2}$	x
x	1
x ²	2x
2x	2
n	0

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

Product Rule: $\frac{d}{dx} [f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

Four Steps from Product Rule to IBP formula

- Integrate all terms in the Product Rule:

$$f(x)g(x) = \int f(x)g'(x)dx + \int g(x)f'(x)$$
- Rewrite in IBP friendly notation such that:

$$f(x) = u \quad f'(x) dx = du$$

$$g(x) = v \quad g'(x) dx = dv$$
- Rewritten: $uv = \int u dv + \int v du$
- Rearrange: $\int u dv = uv - \int v du$ Ta-da! IBP formula!
 (hey, remember to finish evaluations by attaching + C to indefinite integrals)

Tabular Integration: $\int x^4 \sin x dx = ?$

signs u = x⁴ dv = sin x dx

(+) x⁴ → sin x
 (-) 4x³ → -cos x
 (+) 12x² → -sin x
 (-) 24x → cos x
 (+) 24 → sin x
 0 → -cos x

$$= -x^4 \cos x + 4x^3 \sin x + 12x^2 \cos x - 24x \sin x - 24 \cos x + C$$